NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1280

DISTRIBUTION OF BEARING REACTIONS ON A ROTATING SHAFT SUPPORTED ON MULTIPLE JOURNAL BEARINGS

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SUMMARY

An analytical treatment of the problem of determining the distribution of reaction forces among multiple plain bearings supporting a shaft subjected to rotating loads is presented. The treatment differs from others in common use in that account is taken of the hydrodynamic effect of the oil film between the journals and the bearings. A relation is expressed between the eccentric running position of a journal within its bearing and the load hydraulically supported by this bearing; this relation, together with the conditions of elasticity for the shaft and the bearing supports, provides a sufficient number of equations for an analytical determination of the reactions at each bearing of a multiple-bearing system.

An investigation conducted on a straight shaft operating in a seven-bearing crankcase of an aircraft engine is presented to show the importance of the hydrodynamic effect of the oil film. Results indicated that the hydrodynamic effect should be considered in problems involving the distribution of bearing reactions.

INTRODUCTION

A knowledge of the distribution of reactions among the bearings of a dynamically loaded shaft supported at three or more plain journal bearings is of importance in the design of shafts and bearings. The standard design practice of calculating the bearing loads in in-line aircraft engines developed by Caminez and Iseler in 1923 assumes that a load applied between two bearings produces equal reactions at these two bearings and that no reactions are present at remote bearings. Refinements are often considered that take into account the elasticity of the crankshaft and crankcase (references 1 to 3); in reference 4 consideration is given to the mechanical effect of clearance between the journals and the bearings. Another factor that has not been considered is the hydrodynamic factor resulting from the presence of an oil film between the journals and the bearings. This factor may considerably affect the bearing-load distribution.

A journal floating in a plain bearing is supported by the hydraulic pressure of the oil film in the space between the journal and the bearing. The position that the journal assumes in the bearing is dependent, among other factors, on the load carried by the bearing and on the hydrodynamic properties of the oil film. A set of interrelations therefore results among the properties of the oil films in the various bearings and the loads carried by the bearings. The bearing loads may be determined from equations expressing these relations.

An elementary solution of the problem of a shaft supported on seven bearings and loaded between any two of these bearings is presented. The solution is elementary in that ideal properties are assumed for the elastic properties of the shaft and bearing systems and for the hydrodynamic properties of the oil film. Furthermore, a linear relation is assumed between the bearing load and the eccentric running position of the journal in the bearing, a condition that is more likely to be present under light loading of the shaft than under the heavy loading ordinarily encountered, for example, in in-line aircraft engines. The method does, however, show the approach that can be made towards the solution of the problem and it also emphasizes the importance of such variables as clearance over which the designer has some control and whereby the bearing-load distribution may be controlled.

The results of an experimental program undertaken to test the validity of the analysis are also reported. In this program, rotating loads were applied to a straight shaft and to a crankshaft supported in a conventional aircraft-engine seven-bearing crankcase. The load carried by each of the bearings was measured by wire-resistance strain gages so mounted in the bearings as to indicate the hydraulic pressure in the oil film carrying the load. This investigation shows the importance of the hydrodynamic effect of the oil film and the discrepancy that can exist between the conventionally calculated and the measured bearing loads.

TREATMENT OF PROBLEM OF MAIN BEARING LOADS ON BASIS OF HYDRODYNAMIC THEORY OF LUBRICATION

It has been established by the hydrodynamic theory of lubrication (reference 5, pp. 106-125) that a loaded journal floats eccentrically within the oil film in the clearance space between the journal and the bearing and that the amount and the direction of the eccentricity are dependent, among other factors, on the load. For a journal bearing without end leakage, the relation is given by equation (81) of reference 5 (p. 113)

$$P = \frac{\mu U L r^2}{c^2} C_p \tag{1}$$

where

P load, pounds

μ viscosity of lubricating oil, reyns

U peripheral velocity of journal, inches per second

L length of bearing (dimension parallel to bearing axis), inches

r radius of bearing, inches

c radial clearance between journal and bearing, inches

C_p dimensionless load factor dependent upon e/c, as shown in figure 1

e eccentricity of journal (distance between centers of journal and bearing), inches

For moderate values of e/c (less than 0.75), C_p is approximately proportional to e/c; in this case equation (1) reduces to

$$e = kP \tag{2}$$

where

k constant dependent upon value of C_p

For a bearing of infinite axial length, the directions of the load P and of the eccentricity e are at right angles, as shown in figure 2(a). When both the load and the eccentricity are resolved into X and Y components, it is seen that

$$\begin{aligned} e_Y &= kP_X \\ e_X &= -kP_Y \end{aligned} \tag{3}$$

The X component of load is proportional to the Y component of eccentricity; the Y component of load is negatively proportional to the X component of eccentricity.

In consideration of practical bearings of short axial length, equation (1) may be expressed as

$$k'P = \frac{\mu U L r^2}{c^2} C_p \tag{4}$$

The coefficient k' is greater than unity and depends upon the ratio of the effective length in the direction of motion of the bearing (one-half of the bearing circumference) to the axial length of the bearing. (See reference 5, p. 175.) The greater the ratio, the larger will be the coefficient. In addition, as the bearing length L becomes less, the direction of eccentricity e approaches the direction of load P until the direction of eccentricity is nearly coincident with that of the load. This concept is presented in figure 2(b). Actually, the coincidence of these directions can be accomplished only by increasing e/c to high values at which P is not exactly proportional to e. For this simplified consideration, however, the proportionality relation may be expressed by

$$e_X = KP_X \tag{5}$$

where K is a proportionality factor. Equation (5) states that the journal assumes an eccentric position in the bearing and that the eccentricity is in the direction of the load and proportional to it. The proportionality factor K is greater than the factor k of equation (3).

Application of Theory

An unloaded rotating shaft will float concentrically within the bearings, supported by an oil film of uniform thickness in each bearing. As load is applied, the shaft assumes a position of eccentricity with regard to the bearing. This eccentricity may occur because of shaft or bearing displacement. Actually, because both shaft and bearing frame (in this analysis, a straight shaft and seven bearings) are assumed to have elasticity, both types of displacement may occur. Running eccentricity may be considered to be the difference between the displacement of the journal and that of its supporting bearing. At any bearing therefore

$$e_n = \delta_n - \delta_{n'} = KX_n \tag{6}$$

where

e_n running eccentricity, inches

δ_n displacement of journal, inches

δ_n' displacement of bearing, inches

X_n reaction at bearing n when shaft is loaded, pounds

n number indicating location in arbitrary system of numbering bearings with reference to bearing frame

The displacements of the shaft at the locations of the various bearings are first considered. Two types of displacement are possible: rigid-body and shaft-bending deflection displacements. Figure 3 shows the sketches used in the analysis; the horizontal line indicates the position that the center line of the unloaded shaft or bearing frame would assume; the curved line shows the center line that the shaft or frame would have when deflected by load; the dashed line indicates the change in position that would occur if the shaft was displaced in the bearings without bending. In figure 3(a), the displacements of the shaft from the central position at bearings 1 and 7 are taken as δ_1 and δ_7, respectively, for an external load p applied between bearings 6 and 7. If the shaft were perfectly rigid, there would be a displacement at every journal as shown by the dashed line in figure 3(a). For example, at the location of bearing 2 there would be a rigid-body displacement of $\delta_1 + \frac{1}{6}(\delta_7 - \delta_1)$. In addition, there is the displacement of the journal at bearing 2 resulting from the bending produced by the load p and the bearing reactions X_2, X_3, . . . at each of the bearings. The total displacement of the shaft at the location of bearing 2 is

$$\delta_2 = \delta_1 + \frac{1}{6}(\delta_7 - \delta_1) - d_{2,2}X_2 - d_{3,2}X_3 - d_{4,2}X_4 - d_{5,2}X_5 - d_{6,2}X_6 + d_{p,2}\,p \tag{7}$$

where

$d_{n,2}$ simple bending deflection of shaft at bearing 2 caused by unit reaction load at bearing n

$d_{p,2}$ simple bending deflection of shaft at bearing 2 caused by applied load between bearings 6 and 7

The deflections of the bearing frame at the locations of the various bearings are next considered. The bearing frame is assumed fixed at two supports (at bearings 1 and 7) and the only loads applied to it are the loads at the bearings. Displacements consist only of simple bending deflections; for bearing 2 the deflection is given by

$$\delta_2{}' = d_{2,2}{}'X_2 + d_{3,2}{}'X_3 + d_{4,2}{}'X_4 + d_{5,2}{}'X_5 + d_{6,2}{}'X_6 \qquad (8)$$

where

$d_{n,2}{}'$ simple bending deflection of frame at bearing 2 caused by a unit reaction load at bearing n

When equation (8) is subtracted from equation (7)

$$\begin{aligned}\delta_2 - \delta_2{}' = \delta_1 + \frac{1}{6}(\delta_7 - \delta_1) + d_{p,2}P - (d_{2,2} + d_{2,2}{}')X_2 \\ - (d_{3,2} + d_{3,2}{}')X_3 - (d_{4,2} + d_{4,2}{}')X_4 - (d_{5,2} + d_{5,2}{}')X_5 \\ - (d_{6,2} + d_{6,2}{}')X \qquad (9)\end{aligned}$$

The eccentricity is proportional to the load and from equation (6)

$$\delta_2 - \delta_2{}' = KX_2 \qquad (10)$$

$$\delta_1 = KX_1 \qquad (11)$$

$$\delta_7 = KX_7 \qquad (12)$$

If

$$\Sigma_{n,2} = d_{n,2} + d_{n,2}{}'$$

where

$\Sigma_{n,2}$ sum of simple bending deflection of shaft at bearing 2 caused by unit reaction load at bearing n and simple bending deflection of frame at bearing 2 caused by unit reaction load at bearing n

equation (9) reduces to

$$\begin{aligned}KX_2 = KX_1 + \frac{1}{6}K(X_7 - X_1) + d_{p,2}P - \Sigma_{2,2}X_2 - \Sigma_{3,2}X_3 - \Sigma_{4,2}X_4 \\ - \Sigma_{5,2}X_5 - \Sigma_{6,2}X_6 \qquad (13)\end{aligned}$$

A first approximation of the bending deflections $d_{2,2}$, $d_{2,2'}$, $d_{3,2}$, . . . can be obtained if the shaft is assumed to be a uniform beam of rigidity E_1I_1 and the bearing frame a uniform beam of rigidity E_2I_2, where E represents the modulus of elasticity and I the moment of inertia. By use of the notation of figure 3(b) (see formulas of reference 6 (pp. 168-169))

$$d_{2,2} = \frac{D_rD_l}{6DE_1I_1}\left(D^2 - D_l^2 - D_r^2\right) = \frac{(10l)(2l)}{(6)(12l)(E_1I_1)}$$
$$\left[(12l)^2 - (2l)^2 - (10l)^2\right] = \frac{100}{9}\frac{l^3}{E_1I_1} \qquad (14)$$

where

D length of beam, inches

D_l distance between left-hand end of beam and load caused only by deflection, inches

D_r distance between right-hand end of beam and load under consideration, inches

l one-half of distance between adjacent bearings, inches

Similarly, for the frame

$$d_{2,2'} = \frac{100}{9}\frac{l^3}{E_2I_2} \qquad (15)$$

Therefore

$$\Sigma_{2,2} = \frac{100}{9}\left(\frac{l^3}{E_1I_1} + \frac{l^3}{E_2I_2}\right) \qquad (16)$$

By following a similar procedure, the remaining values of $\Sigma_{n,2}$ can be determined. Figure 3(c) shows the notation upon which the definition of the coefficient $d_{p,2}$ is based.

$$d_{p,2} = \frac{D_rD_l}{6DE_1I_1}\left(D^2 - D_r^2 - D_l^2\right) = \frac{l\ (2l)}{(6)(12l)\ E_1I_1}$$
$$\left[(12l)^2 - (l)^2 - (2l)^2\right] = \frac{139}{36}\frac{l^3}{E_1I_1} \qquad (17)$$

When these values are substituted into equation (13) and when the fractions are eliminated from the numerical values

$$139\frac{l^3}{E_1I_1}p = \left(\frac{l^3}{E_1I_1} + \frac{l^3}{E_2I_2}\right)(400X_2 + 608X_3 + 624X_4 + 496X_5 + 272X_6) - 30KX_1 + 36KX_2 - 6KX_7 \tag{18}$$

When equation (18) is divided by $\left(\frac{l^3}{E_1I_1} + \frac{l^3}{E_2I_2}\right)$

$$139\left(\frac{1}{1 + \frac{E_1I_1}{E_2I_2}}\right)p = 400X_2 + 608X_3 + 624X_4 + 496X_5 + 272X_6 + \left(\frac{K}{\frac{l^3}{E_1I_1} + \frac{l^3}{E_2I_2}}\right)(-30X_1 + 36X_2 - 6X_7) \tag{19}$$

Let

$$\omega = \frac{1}{1 + \frac{E_1I_1}{E_2I_2}}$$

and

$$m = \frac{K}{\frac{l^3}{E_1I_1} + \frac{l^3}{E_2I_2}}$$

where

ω constant dependent on moduli of shaft and bearing frame

m stiffness parameter dependent upon moduli of shaft and bearing frame, physical dimensions of shaft and bearing frame, and dimensional and nondimensional quantities given in equation (1)

When these values are substituted into equation (19),

$$139\omega p = -30mX_1 + (400 + 36m)X_2 + 608X_3 + 624X_4 + 496X_5 + 272X_6 - 6mX_7 \tag{20}$$

The equations for the displacements at the remaining intermediate bearing locations 3 to 6 are determined by a similar procedure. In order to obtain the additional equations required for simultaneous solution, an equation for static equilibrium is written

$$p = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 \tag{21}$$

and in addition a moment equation is set up by taking moments about the location of bearing 1. (See figure 3(a).)

$$11p = 2X_2 + 4X_3 + 6X_4 + 8X_5 + 10X_6 + 12X_7 \tag{22}$$

The bearings are assumed to be spaced equally and to be symmetrical with respect to the center main bearing. It is therefore necessary to consider only the successive applications of load between bearings 6 and 7, 5 and 6, and 4 and 5. The equations for the corresponding locations of applied load between bearings 1 and 2, 2 and 3, and 3 and 4 are in the same form.

The following summary of the three sets of simultaneous equations required for solution of a seven-bearing problem completely defines the hydrodynamic analysis:

For a load between bearings 6 and 7

$$p = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 \tag{21}$$

$$11p = 2X_2 + 4X_3 + 6X_4 + 8X_5 + 10X_6 + 12X_7 \tag{22}$$

$$\begin{aligned} 139\omega p = {} & -30mX_1 + (400 + 36m)X_2 + 608X_3 + 624X_4 + 496X_5 \\ & + 272X_6 - 6mX_7 \end{aligned} \tag{20}$$

$$\begin{aligned} 127\omega p = {} & -12mX_1 + 304X_2 + (512 + 18m)X_3 + 552X_4 + 448X_5 \\ & + 248X_6 - 6mX_7 \end{aligned} \tag{23}$$

$$\begin{aligned} 107\omega p = {} & -6mX_1 + 208X_2 + 368X_3 + (432 + 12m)X_4 + 368X_5 \\ & + 208X_6 - 6mX_7 \end{aligned} \tag{24}$$

$$\begin{aligned} 158\omega p = {} & -6mX_1 + 248X_2 + 448X_3 + 552X_4 + (512 + 18m)X_5 \\ & + 304X_6 - 12mX_7 \end{aligned} \tag{25}$$

$$215\omega p = -6mX_1 + 272X_2 + 496X_3 + 624X_4 + 608X_5 + (400 + 36m)X_6 - 30mX_7 \tag{26}$$

For a load between bearings 5 and 6

$$p = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 \tag{21}$$

$$9p = 2X_2 + 4X_3 + 6X_4 + 8X_5 + 10X_6 + 12X_7 \tag{27}$$

$$393\omega p = -30mX_1 + (400 + 36m)X_2 + 608X_3 + 624X_4 + 496X_5 + 272X_6 - 6mX_7 \tag{28}$$

$$357\omega p = -12mX_1 + 304X_2 + (512 + 18m)X_3 + 552X_4 + 448X_5 + 248X_6 - 6mX_7 \tag{29}$$

$$297\omega p = -6mX_1 + 208X_2 + 368X_3 + (432 + 12m)X_4 + 368X_5 + 208X_6 - 6mX_7 \tag{30}$$

$$426\omega p = -6mX_1 + 248X_2 + 448X_3 + 552X_4 + (512 + 18m)X_5 + 304X_6 - 12mX_7 \tag{31}$$

$$531\omega p = -6mX_1 + 272X_2 + 496X_3 + 624X_4 + 608X_5 + (400 + 36m)X_6 - 30mX_7 \tag{32}$$

For a load between bearings 4 and 5

$$p = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 \tag{21}$$

$$7p = 2X_2 + 4X_3 + 6X_4 + 8X_5 + 10X_6 + 12X_7 \tag{33}$$

$$575\omega p = -30mX_1 + (400 + 36m)X_2 + 608X_3 + 624X_4 + 496X_5 + 272X_6 - 6mX_7 \tag{34}$$

$$515\omega p = -12mX_1 + 304X_2 + (512 + 18m)X_3 + 552X_4 + 448X_5 + 248X_6 - 6mX_7 \tag{35}$$

$$415\omega p = -6mX_1 + 208X_2 + 368X_3 + (432 + 12m)X_4 + 368X_5 + 208X_6 - 6mX_7 \tag{36}$$

$$553\omega p = -6mX_1 + 248X_2 + 448X_3 + 552X_4 + (512 + 18m)X_5 + 504X_6 - 12mX_7 \tag{37}$$

$$637\omega p = -6mX_1 + 272X_2 + 496X_3 + 624X_4 + 608X_5 + (400 + 36m)X_6 - 30mX_7 \tag{38}$$

In order to obtain a complete solution of these equations, it would be necessary to consider all possible values of ω and m. Inspection of the equations shows that, for a given value of m, all reactions are linear functions of ω. Solutions therefore can be obtained for $\omega = 1$ (a completely rigid frame) and for $\omega = 0$ (perfectly flexible frame), and a linear interpolation can be made between these solutions for any other value of ω. Figures 4 to 6 show the reactions for the case $\omega = 1$ at each of the bearings for loads applied between any two bearings.

Limitations of Theory

The limitations of the theory should be considered in order to evaluate the extent to which the results can be considered quantitative.

Equation (1) is based on the analytical hydrodynamic theory of lubrication, which assumes a complete and uniform oil film and other ideal conditions not ordinarily realized in practice. The solution proceeding from this equation makes the tacit assumption that the viscosity is uniform within a given bearing and among all bearings. This assumption is open to question because the oil in the bearings taking the highest loads is heated more, which reduces its viscosity. Also, the clearance is assumed constant among all bearings. In practice the clearances are different for all bearings, which must have an important effect on the result because c enters as a squared term in equation (1) and actually exists as a hidden cube term in the factor K of equation (5). The length is also assumed constant for all bearings, which is not necessarily the case in all practical applications.

Another serious limitation is the assumption of equation (5) that the journal assumes an eccentric position in the bearing and that the eccentricity is in the direction of the load and proportional to it. Within the linear portion of the curve of figure 1, equation (3) would probably be more valid. A solution based on equation (3) that involves consideration of reactions at right angles to the applied load and results in 14 equations and 14 unknowns has also been obtained. The results for reaction forces in the plane of the external loads are substantially the same as those determined from the seven-equation solution that is presented. A more serious discrepancy occurs, however, because operation is usually in the region of large values of e/c, for which the curve is not linear.

An additional source of error lies in the calculation of the deflection numbers by the use of equations (14) and (15). In order to attach numerical significance to these coefficients, it is necessary to assume that both the bearing frame and the shaft could be replaced by equivalent uniform beams. The assumption is, of course, open to question.

In view of the foregoing limitations, it is evident that the theory should be considered for its qualitative indications rather than for its quantitative conclusions. Its most significant contribution is the indication of the importance of the oil film on load distribution and of the variables that affect the solution. Equation (1) shows, for example, the importance of amount of clearance. It appears that the distribution of bearing loads can be affected by the relative clearances among bearings. It may, in fact, be desirable to control the loads by the proper proportioning of the clearances.

EXPERIMENTAL METHODS AND EQUIPMENT

A schematic diagram showing the arrangement of the principal parts of the test equipment and a sketch of a modified bearing are presented in figure 7. A crankcase without cylinder blocks from a 12-cylinder aircraft engine was used as a seven-bearing frame for a straight hollow shaft and for a conventional crankshaft. A diagram of the test setup is shown in figure 7(a). Loads were centrifugally applied to the straight shaft by means of unbalanced weights that could be attached at any desired location. Power for rotation of the shaft was obtained from an electric motor through a variable-speed changer.

Qualitative measurements of bearing reactions were made by the use of wire-resistance strain gages one-sixteenth inch in length. As shown in figure 7(b), two strain gages were mounted in the centers of circular recesses in the back of each bearing shell at top-center position, equidistant between the center of the oil-distribution groove and the outer edges of the bearing surface. The strain gages consist of filaments of very fine resistance wire wound on thin forms of nonconducting material. When mounted upon a test member, these gages are insulated from the surface of the member by a thin coating of synthetic resin, which also bonds the gages to the surface. The strain that is produced in the test part by applications of load causes a similar strain in the gage wire with a resultant resistance change. This change in electrical resistance, when suitably interpreted, indicates the magnitude of strain in the part at the location of the gage. The flat surfaces of the circular recesses in the backs of the bearings were considered to form thick diaphragms; the hydraulic pressure within the oil film was considered a load. The maximum strain occurs at the center of a loaded diaphragm; therefore, the loads imposed by the hydraulic pressures within the oil film would cause resistance changes in the strain gages. Strain-gage signals were recorded on an oscillograph. The angular position of the unbalanced weight with reference to strain-gage signals was indicated by a timing mark recorded upon the oscillograph film simultaneously with the gage signals.

The shaft speed was 2000 rpm under the following conditions:

1. An unbalanced weight was placed successively between each pair of adjacent bearings.

2. Four unbalanced weights were so placed between bearings that two were attached to the shaft 180° from the others. Those weights that would exert centrifugal forces in the same direction were located equidistant from the center main bearing. The three possible combinations were all balanced systems.

Similar runs were also made using a conventional crankshaft in place of the straight hollow shaft. The results obtained were essentially similar to those for the straight shaft.

EXPERIMENTAL RESULTS

The test results are summarized in figures 8 and 9. The applied external loads are indicated by solid discontinuous vectors and the amplitude of the observed strain-gage signals by dashed vectors.

The amplitudes of the oscillograph records were used as a criterion of the relative magnitudes of the bearing reaction forces when external loads were applied. The magnitudes of the output signals are only of qualitative value because it was found that increase of signal was not linear with the square of the speed. It is probable that distortions in the shaft and the bearing frame together with the redistribution of oil pressure in the bearings were responsible for the nonlinear relation between load and peak oil pressure immediately over the diaphragm on which a strain gage was mounted. Although the qualitative nature of these data limits their value insofar as magnitudes of reactions are considered, the existence of the reactions and their directions are clearly shown by the experiments. Similar results were obtained in the runs in which a conventional crankshaft was used in place of the hollow straight shaft.

For comparison between the test results presented in figure 8 for a force applied successively between adjacent bearings and the analytical predictions for a rigid crankcase given in figures 4 to 6, a value of the stiffness parameter m approximately equal to 2 will be assumed. The qualitative test data of figure 8(a) for a load between bearings 1 and 2 shows reactions at bearings 1, 2, and 3, all in the same direction, opposed to the load. The predicted values of figure 4 agree insofar as number and direction of reactions are considered. Reactions predicted by the analytical method for other bearings are negligible.

Figure 8(c), which presents the distribution of reaction forces for a load between bearings 2 and 3, shows that reactions were observed in bearings 2, 3, and 4, all in the same direction, opposed to the applied force. The predicted values in figure 5 do not exactly agree with this information. According to the analysis, reaction forces should appear in bearings 1, 2, 3, and 4, all opposed to the applied force, although the predicted value for bearing 1 is of small magnitude. Reactions in other bearings were negligible.

A discrepancy exists between experimental and predicted results for a load applied between bearings 3 and 4 (fig. 8(e). The predicted values of figure 6 indicate reactions for bearings 2, 3, 4, and 5, whereas only bearings 3, 4, and 5 were affected in the tests. Predicted and observed reactions agree, however, in direction; all were opposed to the load.

According to the principle of superposition ordinarily applied when elasticity alone is considered, it would be expected that the bearing reactions produced by several external loads acting in combination would be equal to the sum of the bearing reactions individually produced by the loads acting alone. In the region of operation in which the load is proportional to the running eccentricity (equation (5)), the principle of superposition would hold even when account is taken of the hydrodynamic effect of the oil film; above the region of linearity (the region of relatively high loads in which the curved portion of fig. 1 holds), the principle of superposition does not necessarily apply. The effect produced by several loads acting in combination is not necessarily the sum of the effects individually produced by the loads acting alone.

The experimental results of figure 9 show the nonadditiveness of the individual reactions produced by external loads acting in combination. Superposition of the reactions shown in figure 8(a), (b), (c), and (d) should give the distribution shown in figure 9(a). Reactions should be almost negligible at bearings 2, 3, 5, and 6. Figure 9(a) indicates, however, that reactions of considerable magnitude were present at bearings 3 and 5 when the external loads were combined. The most important disparity between results predicted by the superposition of reaction distributions of figure 8(a), (b), (e), and (f) and results that were actually observed under combined loading (fig. 9(b)) was the absence of reaction at bearings 2 and 6. The error in the prediction of reaction distribution under complex external loading from reactions observed when single external loads are applied is further exemplified in figure 9(c). Bearings 1 and 7 carry loads that would have been predicted to occur in bearings 2 and 6. Figure 9 demonstrates the discrepancy that can exist between the distribution of bearing loads as predicted by simple design theory and the distribution that actually does exist under running operation.

SIGNIFICANCE OF RESULTS

The hydrodynamic effect of the oil film between the journals and the bearings of a shaft supported on several plain bearings may be a factor of considerable importance in the analyses of the loads produced in the various bearings. An elementary approach to a consideration of this complex factor has been presented. The important variables are indicated and it is concluded that some control may be exercised over the bearing loads by a suitable control over these variables.

A suitable variation of relative clearances among bearings may result in a more equal distribution of loads. Equation (1) shows that the clearance is a very important variable. If the clearance is small, even a small displacement of the shaft in the bearing will introduce a considerable bearing load. Thus it would appear that a slight decrease of the clearances in bearings 2 and 6 would cause these bearings to carry some of the load, and in general, that the bearing loads can to some extent be controlled by control of the relative clearances. The precise type of variation that would produce the most beneficial results would have to be determined by additional analyses and experimentation. In varying the relative clearances, consideration must be given to the fact that a change in clearance also affects the load-carrying capacity of a bearing as well as the load, and that the final effect must be such as to maintain a safe load capacity in each bearing.

Flight Propulsion Research Laboratory,
National Advisory Committee for Aeronautics,
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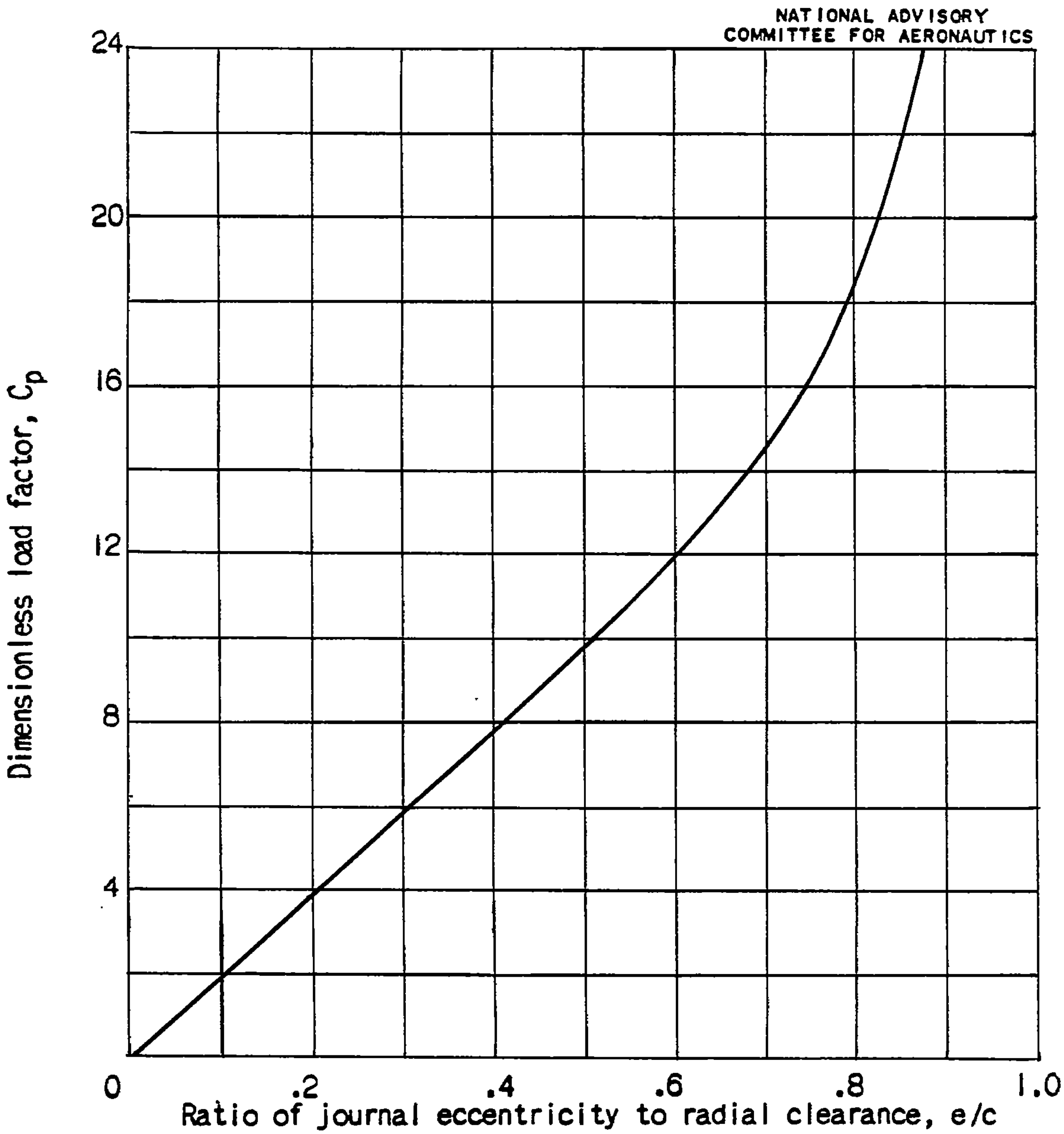
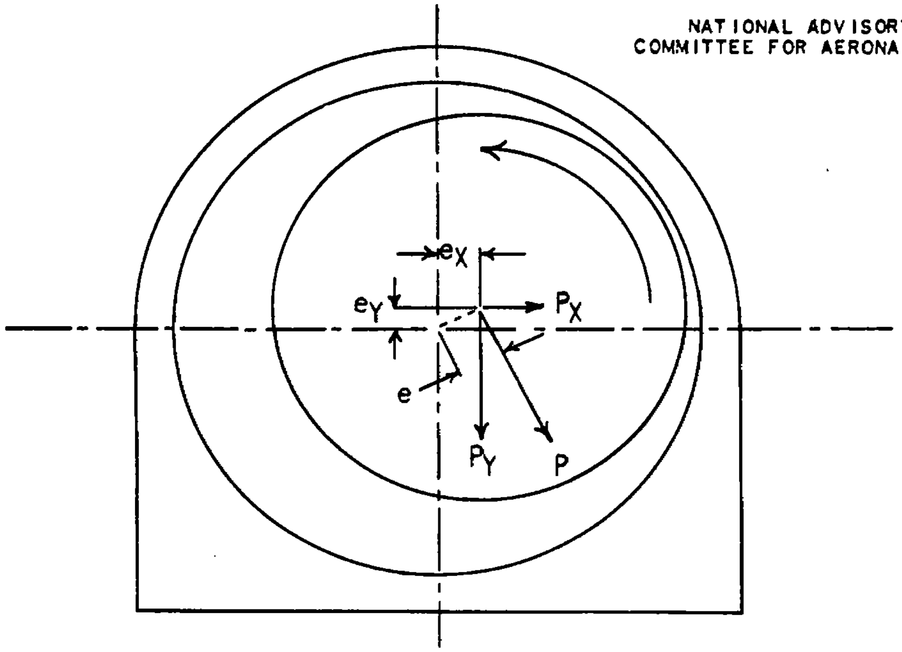
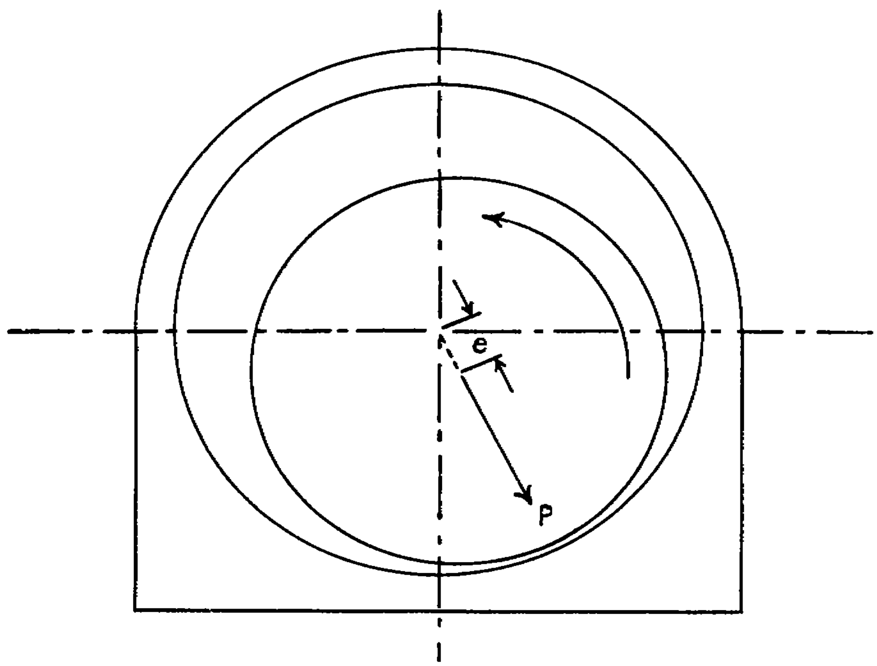


Figure 1. - Load factor for full journal bearing with uniform viscosity and no end leakage.

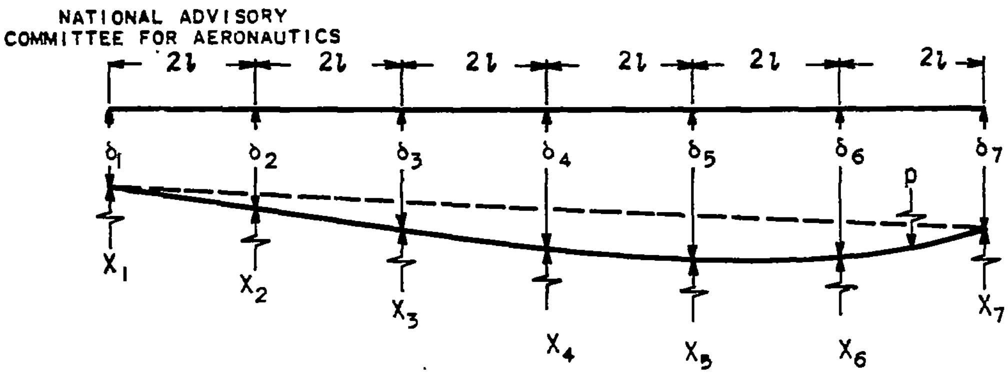


(a) Bearing of infinite axial length.

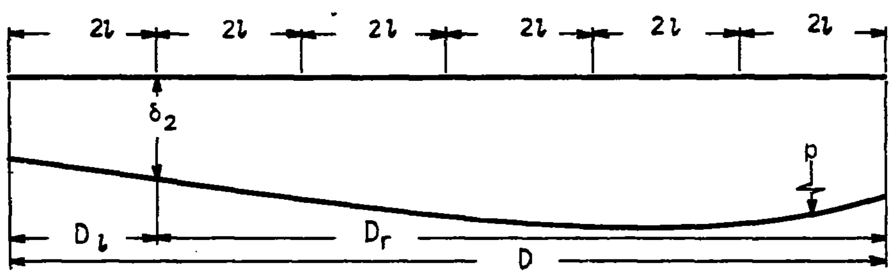


(b) Bearing of short axial length.

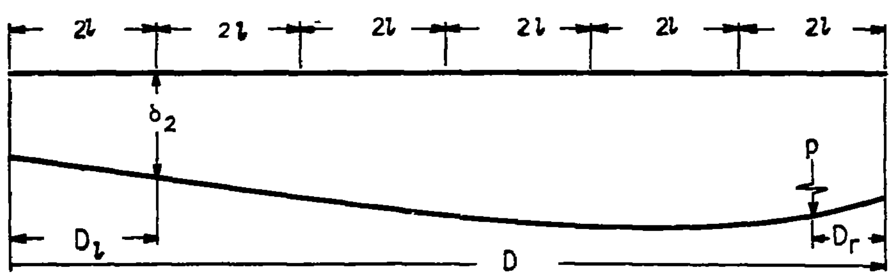
Figure 2. - Representations of journal and bearing with directions of load and eccentricity shown.



(a) Rigid-body displacements and shaft-bending deflection displacements.



(b) Figure used for determination of bending deflections $d_{2,2}$ and $d_{2,2'}$.



(c) Figure used for determination of coefficient $d_{p,2}$.

Figure 3. - Sketches showing dimensions and symbols used in derivation of analysis in case of a shaft on seven bearings with load applied between bearings 6 and 7.

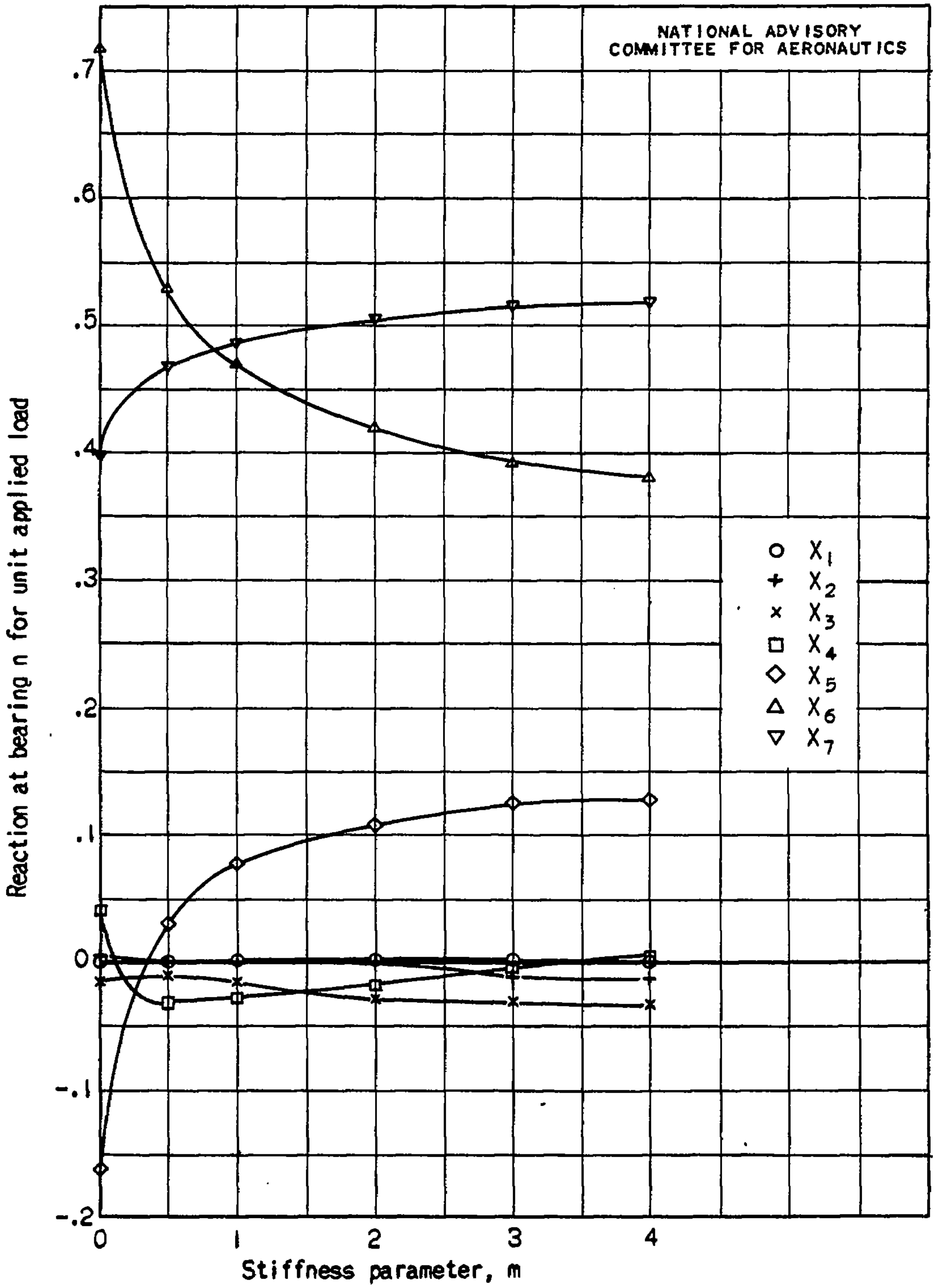


Figure 4. - Relative values of bearing reactions for various values of m when ω = 1 with load applied between bearings 6 and 7. (For load applied between bearings 1 and 2, reverse order of symbols.)

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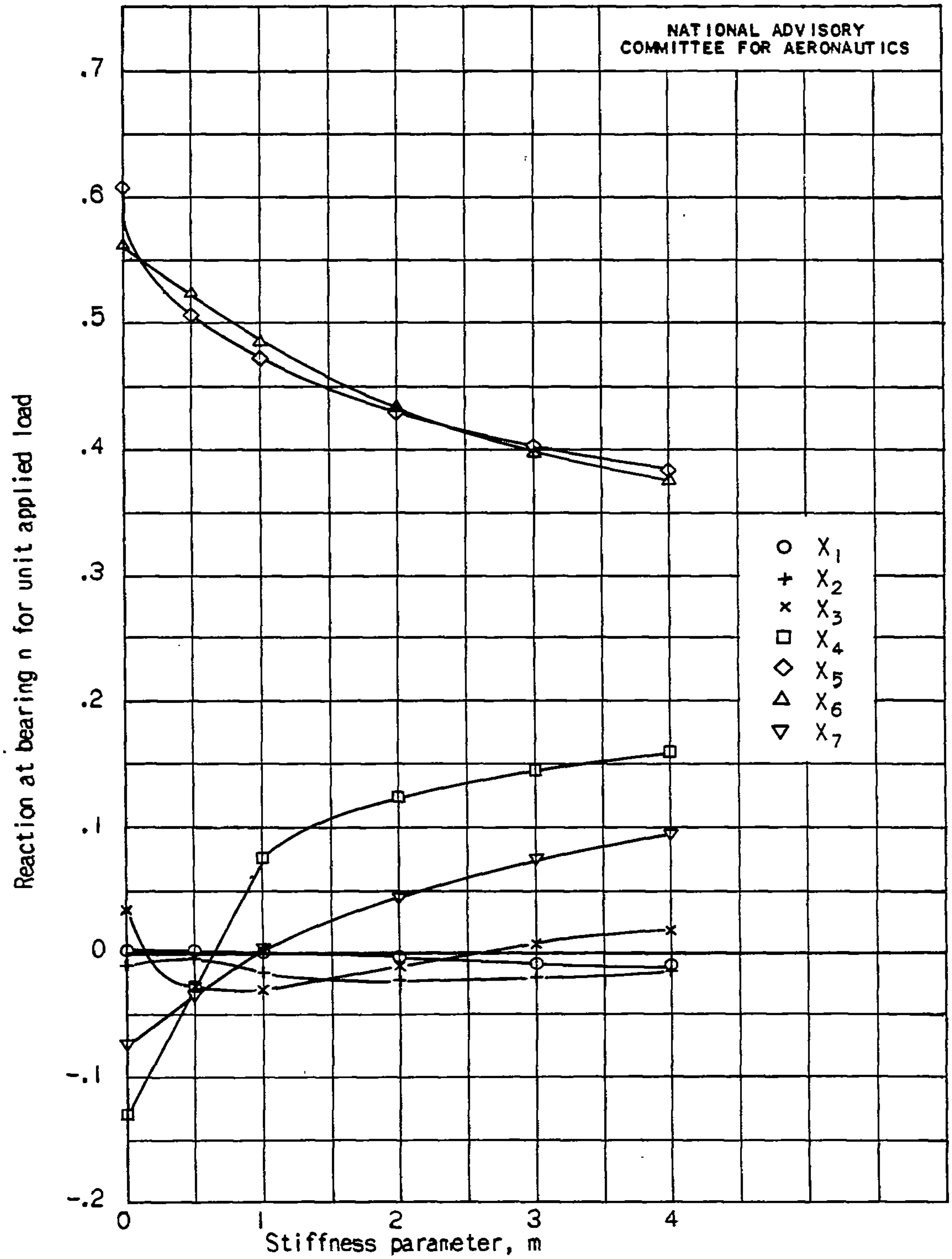


Figure 5. - Relative values of bearing reactions for various values of m when ω = 1 with load applied between bearings 5 and 6. (For load applied between bearings 2 and 3, reverse order of symbols.)

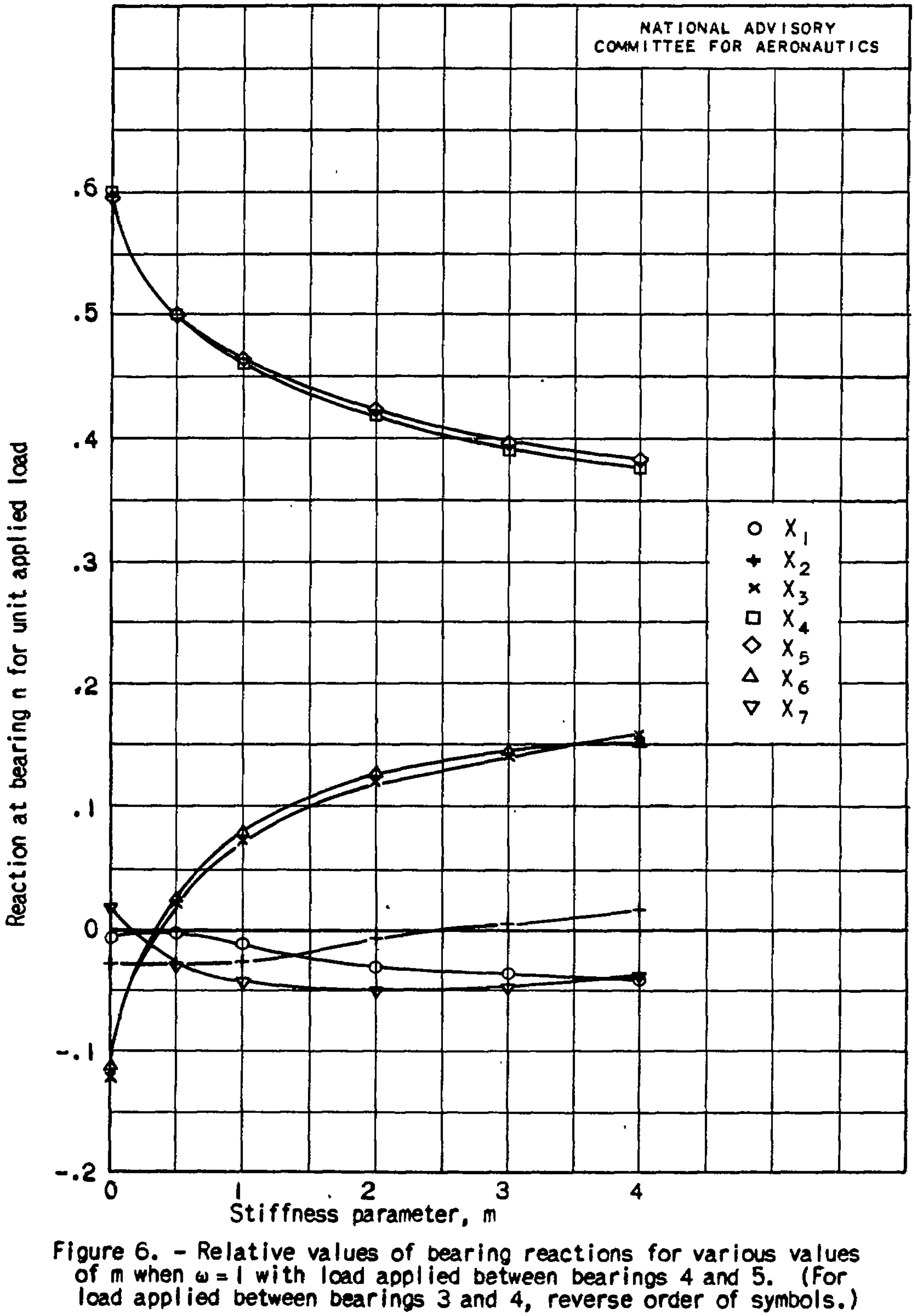
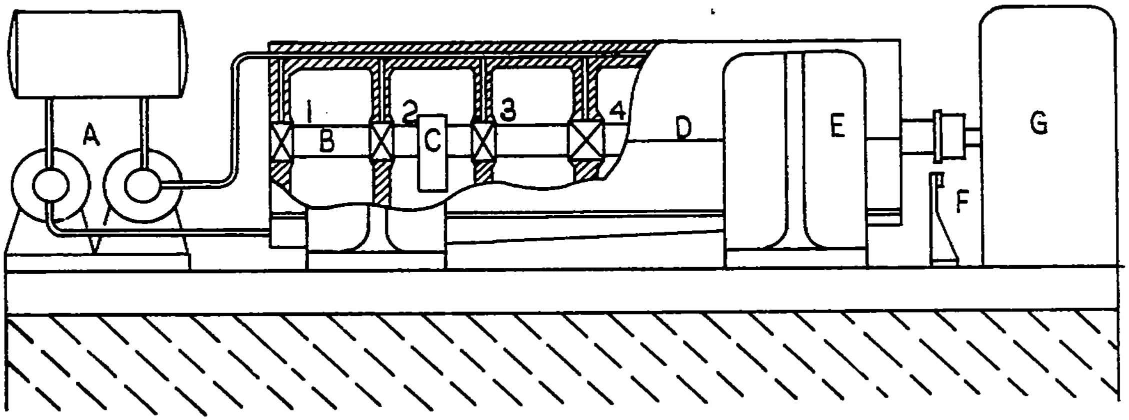


Figure 6. - Relative values of bearing reactions for various values of m when $\omega = 1$ with load applied between bearings 4 and 5. (For load applied between bearings 3 and 4, reverse order of symbols.)



A Oil pumping system
B Straight hollow shaft
C Unbalanced rotating weight
D Bearing frame (crankcase)
E Supporting pedestal
F Timing device
G Electric motor

(a) Schematic diagram of installation. (Sectional view shows bearing numbers according to conventional usage.)

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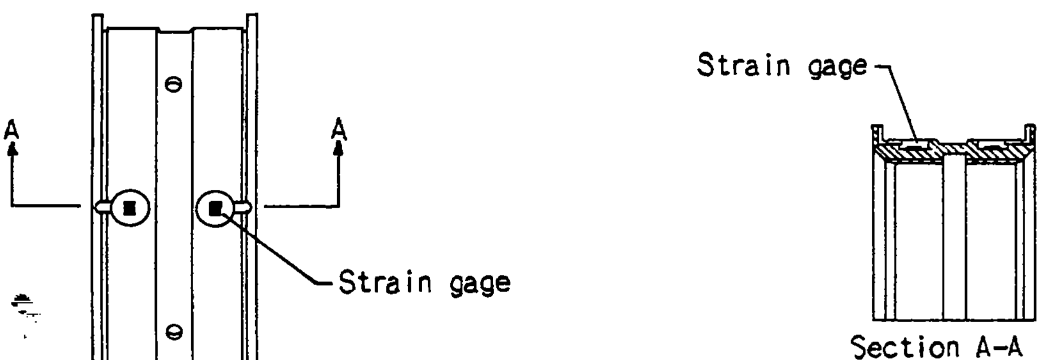

(b) Upper half of bearing shell showing modifications.

Figure 7. - Installation and bearing used in experimental investigation.

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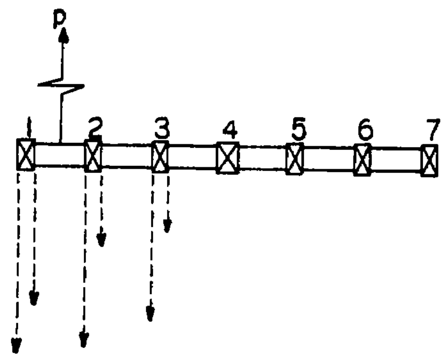
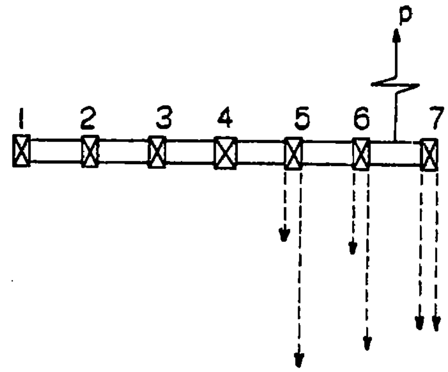
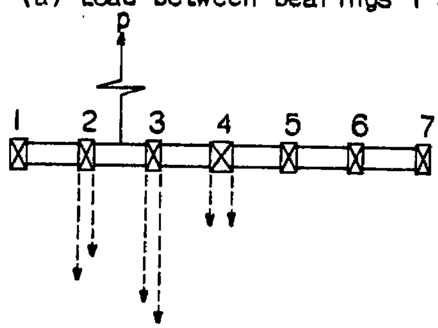


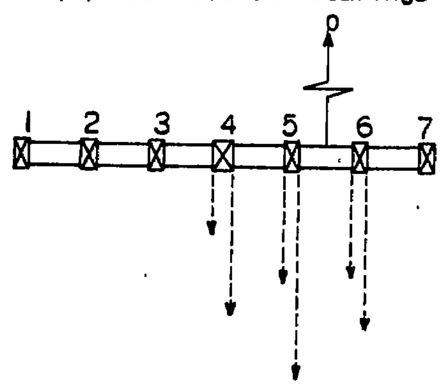
(a) Load between bearings 1 and 2.



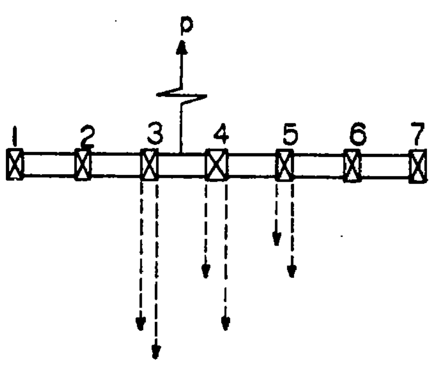
(b) Load between bearings 6 and 7.



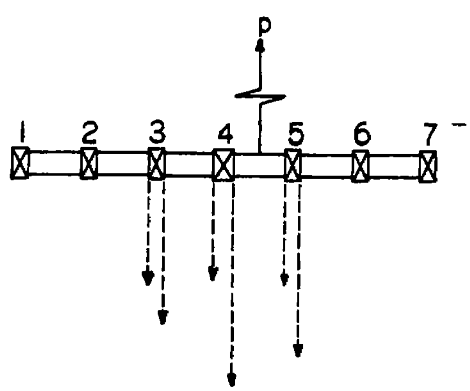
(c) Load between bearings 2 and 3.



(d) Load between bearings 5 and 6.



(e) Load between bearings 3 and 4.



(f) Load between bearings 4 and 5.

Figure 8. - Qualitative distribution of observed reaction forces among seven bearings when a single load p was applied at various locations on the shaft.

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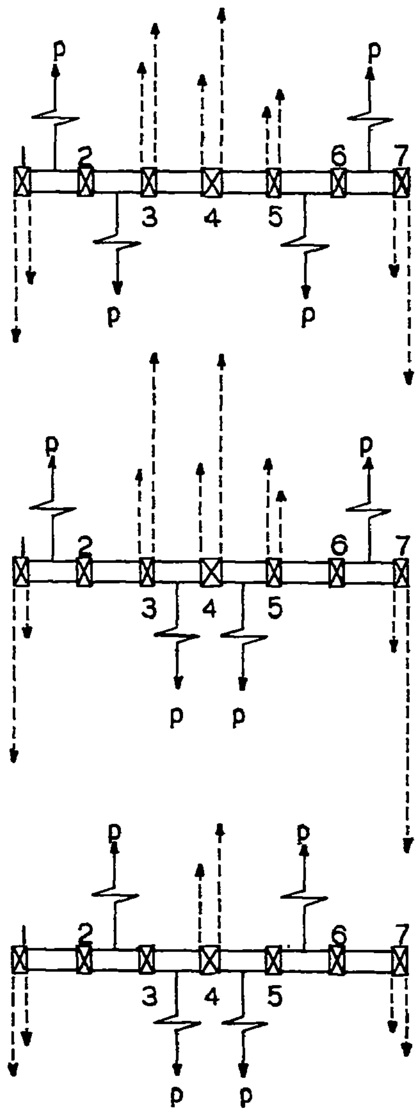

(a) Loads between bearings 1 and 2, 2 and 3, 5 and 6, 6 and 7.

(b) Loads between bearings 1 and 2, 3 and 4, 4 and 5, 6 and 7.

(c) Loads between bearings 2 and 3, 3 and 4, 4 and 5, 5 and 6.

Figure 9. - Qualitative distribution of observed reaction forces among seven bearings when four equal loads p were simultaneously applied on shaft in various combinations.